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THE AREA LAW IN MATRIX MODELS FOR LARGE N QCD STRINGS

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We study the question whether matrix models obtained in the zero volume limit of 4d Yang–Mills theories can describe large N QCD strings. The matrix model we use is a variant of the Eguchi–Kawai model in terms of Hermitian matrices, but without any twists or quenching. This model was originally proposed as a toy model of the IIB matrix model. In contrast to common expectations, we do observe the area law for Wilson loops in a significant range of scale of the loop area. Numerical simulations show that this range is stable as N increases up to 768, which strongly suggests that it persists in the large N limit. Hence the equivalence to QCD strings may hold for length scales inside a finite regime.

Keywords: Large N gauge theory; matrix models; Monte Carlo simulation.

1. Eguchi−Kawai Equivalence

In quantum field theories and systems in statistical mechanics, the number of internal degrees of freedom often enters as a free parameter, which we denote by N. In many cases, the dynamics of the system simplifies significantly as N becomes large. A general discussion is given for instance in Ref. 1.

This effect motivated in particular the model suggested by Eguchi and Kawai a long time ago.[2] Their point of departure was the standard $U(N)$ lattice gauge theory. Based on the factorization of correlation functions at $N \to \infty$, they suggested that the model should be equivalent to its dimensional reduction to one point. Then all the link variables are replaced by $U_{x,\mu} \to U_\mu$, and the plaquette action reduces to:

$$S_{\rm EK} = -N\beta \sum_{\mu\neq\nu=1}^{d} {\rm Tr}(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)\,. \qquad (1)$$

As their argument for the *Eguchi–Kawai equivalence* to ordinary gauge theory, the authors showed that the Schwinger–Dyson equations remain unaltered, which also implies the invariance of the Wilson loops. However, their derivation implicitly assumed that the $[U(1)]^d$ symmetry of the phases is not spontaneously broken. This is correct at strong coupling in dimension >2 and for all couplings in two dimensions. On the other hand, it turned out that at weak coupling in dimension >2, spontaneous symmetry breaking does set in. Numerical simulations revealed that the N eigenvalues of U_μ tend to be almost equal.[3] Hence this attempt to prove general equivalence to standard lattice gauge theory failed, and therefore the equivalence to the continuum gauge theory is uncertain as well.

In order to avoid this problem, the modified versions of Eguchi–Kawai model were introduced, in particular the "quenched Eguchi–Kawai model"[3] and the "twisted Eguchi–Kawai model",[4] where the phase symmetries are not spontaneously broken. Reference 3 also contains numerical results on the quenched Eguchi–Kawai model, and Refs. 5 and 6 present a perturbative consideration. For numerical studies and generalizations of the twisted Eguchi–Kawai model, see for instance Ref. 7.

More recently, Ishibashi, Kawai, Kitazawa and Tsuchiya worked out a large N reduced matrix model of 10d super Yang–Mills theory[8] (for a review, see Ref. 9). The authors presented some evidence for its relation to type IIB superstring theory and conjectured that it could provide a constructive (nonperturbative) definition of string theory. In particular, they showed that the matrix model action reduces to the Green–Schwarz–Schild action in a certain semiclassical limit.[a] Moreover, their *IIB matrix model* is shown to describe D-branes and their interactions correctly, which was further elaborated in Ref. 11.

Supersymmetry seems to protect the $[U(1)]^d$ symmetry,[8] but it turns out to be marginally broken.[12] It is completely broken in the simplified *bosonic IIB matrix model*, which is obtained by omitting the fermions. Nevertheless, the bosonic model has been recognized as a well-defined model,[13] and it has attracted considerable interest in the literature. Both, the bosonic and the supersymmetric model can be formally obtained from the zero-volume limit of large N gauge theories in the continuum, and hence they can be viewed as variants of the original Eguchi–Kawai model. The finiteness of the partition function becomes a nontrivial issue, since the matrices to be integrated are Hermitian. The condition for the finiteness ($N \geq 2$, $d = 4$, 6, 10 for the supersymmetric case, and $N > (3d-4)/(2(d-2))$ for the bosonic case) conjectured in Refs. 13 and 14 has been proved recently.[15] However, the validity of Eguchi–Kawai equivalence is still an open question.

[a] A more in depth study of this correspondence has been reported in Ref. 10.

In particular, the $4d$ case of this Hermitian matrix model is accessible for numerical studies. This is true even for its supersymmetric form, because the fermionic determinant is real positive in $d = 4$,[16] as suspected earlier.[14,b] Large N factorization is known to hold in the $4d$ IIB matrix model, both, for the bosonic[17] and for the supersymmetric case.[16] As a stronger criterion for Eguchi–Kawai equivalence to ordinary gauge theory, one may investigate the validity of the area law for the Wilson one-point function. Simulations for N up to 48 show a finite range of the scale, where the area law seems to hold.[16,c] For the SUSY case, larger N can hardly be simulated on present computers, but in the bosonic case it is possible to go far beyond.[d] The goal of this work is to investigate the fate of the finite area law window as N becomes really large. We increase N up to 768, and observe that the scaling window does indeed stabilize; this provides strong evidence that it does neither shrink to zero nor extend to a larger regime in the large N limit. We also confirm for N up to 768 that the correlation functions relevant for QCD and string theory remain finite in the large N limit if one keeps the product g^2N fixed.[16]

In Sec. 2 we describe the model we are studying, and the properties which are relevant in this context. In Sec. 3 we recall the notion of Wilson loops in the framework of matrix models and their scaling behavior. Then we present our results for the area law at $N = 48$, 128, 256, 512 and 768 for the bosonic $4d$ IIB matrix model. In Sec. 4 we discuss the conclusions from these results, which were mentioned before in Ref. 20.

2. The Model

In 1997, Ishibashi, Kawai, Kitazawa and Tsuchiya suggested a model of supersymmetric matrices, which is a candidate for a nonperturbative definition of IIB superstring theory.[8] Here we are interested in the bosonic variant of this model, which is obtained if one simply drops the fermions by hand. The resulting *bosonic IIB matrix model* is given by the partition function

$$\begin{aligned} Z &= \int dA \, \exp(-S[A]) \,, \\ S[A] &= -\frac{1}{4g^2} \, \mathrm{Tr}([A_\mu, A_\nu]^2) \,, \end{aligned} \tag{2}$$

where A_μ are traceless Hermitian $N \times N$ matrices, and in the d dimensional version of this model, μ runs from 1 to d. The original model was formulated in $d = 10$,

[b] The bosonic model has also been simulated in higher dimensions. In particular, Ref. 17 presents results up to $d = 20$, $N = 32$, which are compared to a $1/d$ expansion, and Ref. 18 arrives at $d = 10$, $N = 128$. Also the SUSY model without fermionic phase factor was simulated in $d = 6$ and $d = 10$.[19]

[c] A similar behavior was also observed in the $10d$ bosonic model.[18]

[d] The computational effort grows like N^5 in the SUSY case, but only like N^3 in the bosonic model.[16]

but here we consider $d = 4$. The constraint of tracelessness is incorporated in the measure,

$$dA = \prod_{\mu=1}^{d} \left[\prod_{i>j} \{ d\,\mathrm{Re}(A_\mu)_{ij}\, d\,\mathrm{Im}(A_\mu)_{ij} \} \prod_{i=1}^{N} d(A_\mu)_{ii}\, \delta \left(\sum_{i=1}^{N} (A_\mu)_{ii} \right) \right] . \tag{3}$$

Note that this model is well-defined without any cutoff for[13,15]

$$N > \frac{3d-4}{2(d-2)} .$$

Hence its only parameter g is a pure scaling parameter, rather than a coupling constant. It can simply be absorbed by rescaling the variables as:

$$A_\mu = \sqrt{g}\, X_\mu , \tag{4}$$

where the matrices X_μ are dimensionless.

This model is invariant under Euclidean rotations of the A_μ, and it has the SU(N) symmetry

$$A_\mu \to V A_\mu V^\dagger , \quad V \in \mathrm{SU}(N) . \tag{5}$$

This $4d$ bosonic IIB matrix model is closely related to the original Eguchi–Kawai model in the weak coupling limit (for details, see, e.g., Ref. 17). Due to the $[U(1)]^d$ spontaneous symmetry breaking the eigenvalues of the model (1) collapse to a point and we obtain

$$S_{\mathrm{EK}} \approx -\frac{1}{2} N\beta \sum_{\mu\neq\nu=1}^{d} \mathrm{Tr}([A_\mu, A_\nu]^2) + \mathcal{O}(A^6) , \tag{6}$$

where $U_\mu = \mathrm{e}^{iA_\mu}$. Thus we can identify $\beta = 1/(2g^2N)$ and the two models agree in the limit $\beta \to \infty$.

Moreover, the large N limit of the bosonic model is expected to be equivalent to the 't Hooft limit of the corresponding Yang–Mills theory with a certain 't Hooft coupling constant λ_{YM} and a momentum cutoff Λ.[17] In this correspondence, however, the 't Hooft coupling constant is related to the momentum cutoff canonically, and therefore one cannot tune λ_{YM} appropriately as one sends $\Lambda \to \infty$.

In the next section we are going to take a new look at the possibility of the Eguchi–Kawai equivalence of the bosonic model to ordinary gauge theory at large N. After all, the presence of spontaneous symmetry breaking does not rule out that Eguchi–Kawai equivalence could be realized for some range of length scales. As a powerful criterion to test this possibility, we study question how far the Wilson loops follow an area law.

3. The Area Law

In analogy to ordinary gauge theory, Polyakov lines and Wilson loops are introduced as:

$$P(k) = \frac{1}{N} \mathrm{Tr}[\exp(ikX_1)] , \tag{7}$$

and

$$W(k) = \frac{1}{N} \mathrm{Tr}[\exp(ikX_1)\exp(ikX_2)\exp(-ikX_1)\exp(-ikX_2)]\,, \tag{8}$$

respectively. For convenience we insert particular components of the dimensionless matrices X_μ, but their choice is irrelevant. In the interpretation of large N reduced matrix models as string theory, Wilson loops play the role of string creation or annihilation operators.[21] The parameter k represents the momentum distribution carried by each section of the string. Its physical (dimensionful) counterpart is given by $k_{\rm phys} = k/\sqrt{g}$. Correlation functions of these operators have a direct physical meaning, and hence the existence of a nontrivial large-N limit is absolutely crucial. It has been observed to exist if one tunes $g \propto 1/\sqrt{N}$, in the bosonic and in the supersymmetric case.[16] In what follows, we set $g = \sqrt{48/N}$ following the convention in Ref. 16. In the Eguchi–Kawai equivalence, $\langle W(k)\rangle$ corresponds to the Wilson loop in the large N gauge theory, and the parameter $k_{\rm phys}$ represents the linear extent of the Wilson loop in a physical scale.

In Fig. 1 we illustrate the large N scaling further by showing the Polyakov line $\langle P\rangle$ as a function of $k_{\rm phys}$, with $N = 48$, 128, 256, 512 and 768. Scaling is confirmed in an impressive way. We also observe that the $k_{\rm phys}$-dependence of the scaling function is in qualitative agreement with an analytical prediction by Oda and Sugino, which is based on a Gaussian approximation (to the second order) in the large N limit.[22] The observation that $\langle P\rangle$ does not vanish at finite values of $k_{\rm phys}$ confirms that the $[U(1)]^4$ symmetry is broken, as we mentioned in Sec. 1.

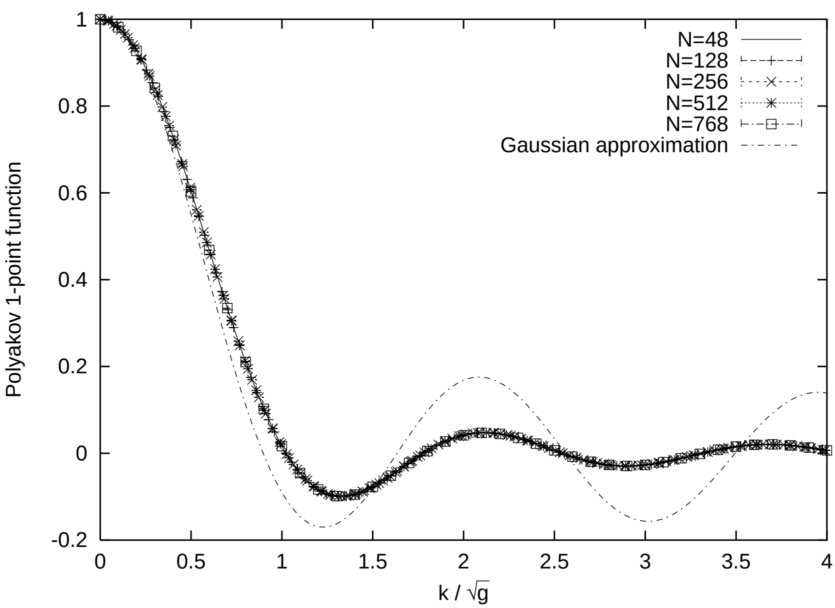

Fig. 1. The Polyakov line $\langle P\rangle$ plotted against the dimensionful momentum $k_{\rm phys} = k/\sqrt{g}$. Numerical results for $N = 48\ldots768$ confirm the large N scaling, as well as qualitative agreement with the Gaussian prediction.

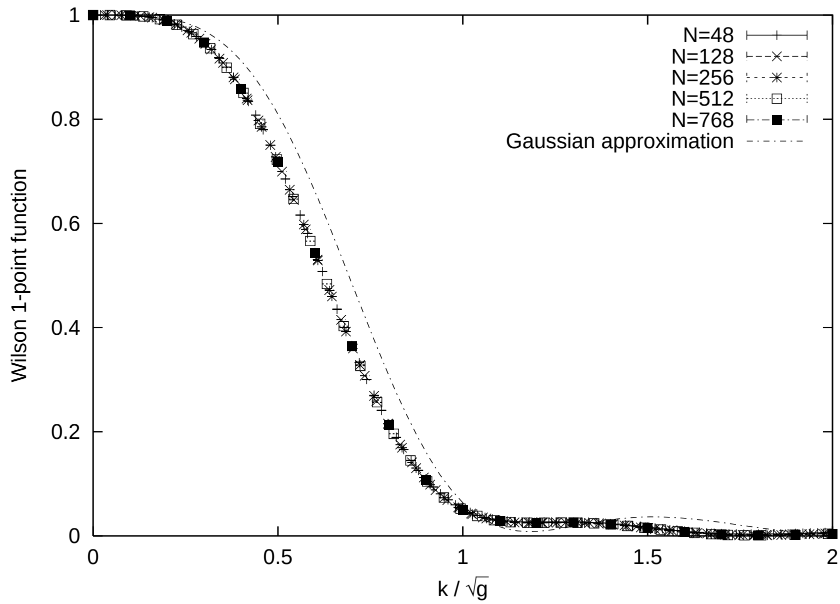

Fig. 2. The Wilson loop $\langle W \rangle$ plotted against the dimensionful momentum $k_{\rm phys} = k/\sqrt{g}$. Numerical results for $N = 48 \ldots 768$ confirm the large N scaling. Again the Gaussian approximation is shown for comparison.

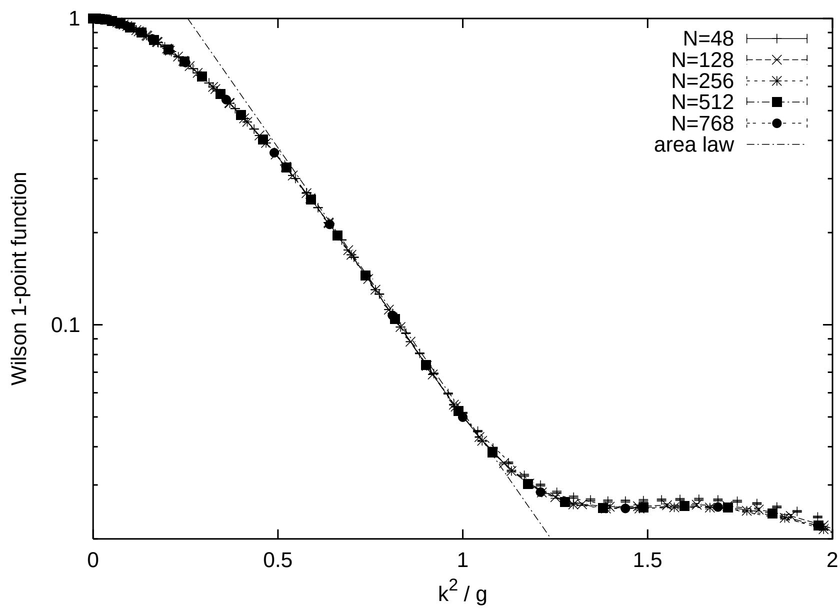

Fig. 3. The Wilson loop plotted logarithmically against $k_{\rm phys}^2 = k^2/g$. The regime $k_{\rm phys}^2 \approx 0.5 \ldots 1$ is consistent with an area law.

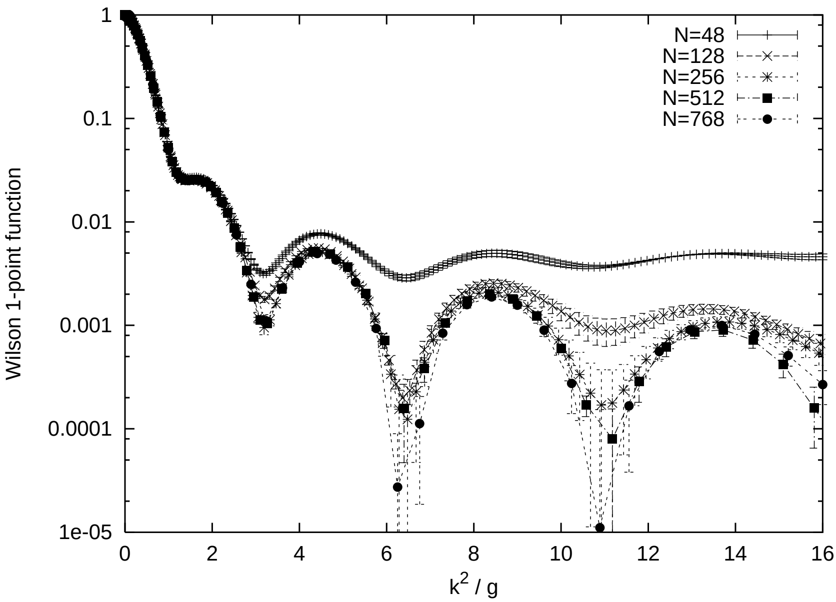

Fig. 4. The Wilson loop plotted against $k^2_{\rm phys} = k^2/g$. There seems to be a sequence of linearity windows at large N, but the relevant area law window is the first one, $k^2/g \approx 0.5 \dots 1$.

Figure 2 is the corresponding plot for the Wilson loop $\langle W \rangle$, also as a function of $k_{\rm phys}$. We see that large N scaling is confirmed again, hence we can take a rather safe extrapolation to $N \to \infty$. Again we show the Gaussian approximation (to the first order)[22] for comparison.

In Fig. 3 we show the Wilson loop logarithmically against $k^2_{\rm phys}$, and we see a window, ranging approximately from $k^2_{\rm phys} \approx 0.5 \dots 1$, which is in agreement with the area law. This area law regime turns out to be manifestly stable as N becomes large. Therefore, we do observe a finite range of scale where Eguchi–Kawai equivalence *is* in business.

In Fig. 4 we extend this plot up to quite large momenta in order to check if perhaps a further area law regime shows up. We observe an oscillating behavior, which also persists at large N. These oscillations might include additional, small area law regimes, but the relevant interval with this respect is clearly the first one, shown in Fig. 3. We also looked for a regime where the perimeter law holds, but we observed that there is no such interval with a size comparable to the leading area law regime.

4. Conclusions

In view of the literature on the Eguchi–Kawai model, it comes as a surprise that the area law does hold in a significant range of scale for the bosonic IIB matrix model. It implies that Eguchi–Kawai equivalence may hold in this regime, even in the absence of twists or other mechanisms to preserve the phase symmetry.

This observation opens up interesting perspectives, with respect to both, the theoretical background and applications. It is now motivated to reconsider the possibility of Eguchi–Kawai equivalence also in the original Eguchi–Kawai model, or further variants thereof, which do not involve a symmetry preserving mechanism. In the matrix model studied here one could still investigate whether the area law regime extends to infinity if we modify the model by the quenching procedure.[3,6] This question is nontrivial given that the Hermitian matrix model has no direct link to lattice gauge theory, and its Eguchi–Kawai equivalence has been discussed only at the perturbative level.[6] Such a study may also shed light on the string-theoretical meaning of the quenching procedure.

We hope that our results presented here provide new insight towards a string description of large N QCD.

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References

1. S. R. Das, *Rev. Mod. Phys.* **59**, 235 (1987).
2. T. Eguchi and H. Kawai, *Phys. Rev. Lett.* **48**, 1063 (1982).
3. G. Bhanot, U. M. Heller, and H. Neuberger, *Phys. Lett. B* **113**, 47 (1982); *Phys. Lett. B* **115**, 237 (1982); U. M. Heller and H. Neuberger, *Nucl. Phys. B* **207**, 399 (1982).
4. A. González-Arroyo and M. Okawa, *Phys. Lett. B* **120**, 174 (1983); *Phys. Rev. D* **27**, 2397 (1983).
5. G. Parisi, *Phys. Lett. B* **112**, 463 (1982).
6. D. J. Gross and Y. Kitazawa, *Nucl. Phys. B* **206**, 440 (1982).
7. A. González-Arroyo and C. P. K. Altes, *Phys. Lett. B* **131**, 396 (1983); K. Fabricius and O. Haan, *Phys. Lett. B* **131**, 399 (1983); *Phys. Lett. B* **139**, 293 (1984); K. Fabricius, O. Haan, and T. Filk, *Phys. Lett. B* **144**, 240 (1984); J. Nishimura, *Mod. Phys. Lett. A* **11**, 3049 (1996), hep-lat/9608119.
8. N. Ishibashi, H. Kawai, Y. Kitazawa, and T. Tsuchiya, *Nucl. Phys. B* **498**, 467 (1997), hep-th/9612115.
9. H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya, and T. Tada, *Prog. Theor. Phys. Suppl.* **134**, 47 (1999), hep-th/9908038.
10. K. N. Anagnostopoulos, J. Nishimura, and P. Olesen, *JHEP* **0104**, 024 (2001), hep-th/0012061; N. Kitsunezaki and S. Uehara, *JHEP* **0110**, 033 (2001), hep-th/0108181.
11. I. Chepelev, Y. Makeenko, and K. Zarembo, *Phys. Lett. B* **400**, 43 (1997), hep-th/9701151; A. Fayyazuddin, Y. Makeenko, P. Olesen, D. J. Smith, and K. Zarembo, *Nucl. Phys. B* **499**, 159 (1997), hep-th/9703038.

12. H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, and T. Tada, *Prog. Theor. Phys.* **99**, 713 (1998), hep-th/9802085.
13. W. Krauth and M. Staudacher, *Phys. Lett. B* **435**, 350 (1998), hep-th/9804199.
14. W. Krauth, H. Nicolai, and M. Staudacher, *Phys. Lett. B* **431**, 31 (1998), hep-th/9803117.
15. P. Austing and J. F. Wheater, *JHEP* **0102**, 028 (2001), hep-th/0101071; *JHEP* **0104**, 019 (2001), hep-th/0103159.
16. J. Ambjørn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, *JHEP* **0007**, 013 (2000), hep-th/0003208.
17. T. Hotta, J. Nishimura, and A. Tsuchiya, *Nucl. Phys. B* **545**, 543 (1999), hep-th/9811220.
18. S. Horata and H. S. Egawa, *Phys. Rev. Lett.* **86**, 4455 (2001), hep-lat/0010051.
19. J. Ambjørn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, *JHEP* **0007**, 011 (2000), hep-th/0005147.
20. J. Ambjørn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta, and J. Nishimura, *Nucl. Phys. B* (*Proc. Suppl.*) **94**, 685 (2001), hep-lat/0009030; "Monte Carlo studies of the dimensionally reduced $4d$ SU(N) super Yang–Mills theory," hep-th/0101084, to appear in the *Proceedings of HEP 2000*, Ioannina, Greece.
21. M. Fukuma, H. Kawai, Y. Kitazawa, and A. Tsuchiya, *Nucl. Phys. B* **510**, 158 (1998), hep-th/9705128.
22. S. Oda and F. Sugino, *JHEP* **0103**, 026 (2001), hep-th/0011175.