Grand Unified Theories and Beyond

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- Antonio Pich: Particle Physics: The Standard Model (lectures)
- W. Hollik: Theory of Electroweak Interactions, Corfu Summer Institute, School and Workshop on Standard Model and Beyond, 2013
- Aitchison I J R & Hey A J G: Gauge Theories In Particle Physics Volume 1: From Relativistic Quantum Mechanics To QED
- Francis Halzen-Alan D.Martin:Quarks and Leptons
- Tai-Pei Cheng,Ling-Fong Li: Gauge Theory of Elementary Particle Physics
- Abdelhak Djouadi: The Anatomy of Electro-Weak Symmetry Breaking, Tome I: The Higgs boson in the Standard Model
- K. Vagionakis: Particle Physics: An Introduction to the Basic Structure of Matter

Standard Model very successful $\rightarrow$ low energy accessible part of a (more) Fundamental Theory of Elemental Particles.
BUT with

- ad hoc Higgs sector
- ad hoc Yukawa couplings

$\rightarrow$ free parameters (> 20)

Renormalization $\rightarrow$ free parameters

Traditional way of reducing the number of parameters:

SYMMETRY

Celebrated example: GUTs

-e.g.Minimal SU(5):

- $\sin^2\theta_w$ (testable)
- m_τ/m_b (successful)

However more SYMMETRY (e.g. $SO(10), E_6, E_7, E_8$) does not necessarily lead to more predictions for the SM parameters.

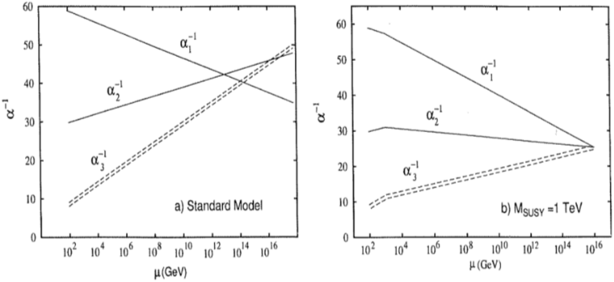
Extreme case: Superstring Theories

On the other hand:

LEP data $\rightarrow$ $N = 1$ $SU(5)$

~~$N = 1$~~ ~~$SU(5)$~~ $\rightarrow$ MSSM

MSSM best candidate for physics beyond SM.



But with > 100 free parameters mostly in its SSB sector.

- Cures problem of quadratic divergencies of the SM (hierarchy problem).
- Restricts the Higgs sector leading to approximate prediction of the Higgs mass.

SM with two Higgs doublets

$$
\begin{aligned}
V &= -\frac{1}{2}m_1^2(H_1^\dagger H_1) - \frac{1}{2}m_2^2(H_2^\dagger H_2) - \frac{1}{2}m_3^2(H_1^\dagger H_2 + h.c.) \\
&+ \frac{1}{2}\lambda_1(H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2(H_2^\dagger H_2)^2 \\
&+ \lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\
&+ \left[\frac{1}{2}\lambda_5(H_1 H_2)^2 + [\lambda_6(H_1^\dagger H_1) + \lambda_7(H_2^\dagger H_2)](H_1^\dagger H_2) + h.c.\right]
\end{aligned}
$$

Supersymmetry provides tree level relations among couplings:

$$
\lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2)
$$

$$
\lambda_3 = \frac{1}{4}(g^2 - g'^2), \quad \lambda_4 = -\frac{1}{4}g^2
$$

$$
\lambda_5 = \lambda_6 = \lambda_7 = 0
$$

with $\upsilon_1 =< ReH_1^0 >$, $\upsilon_2 =< ReH_2^0 >$ and $\upsilon_1^2 + \upsilon_2^2 = (246\ Gev)^2$,$\upsilon_2/\upsilon_1 \equiv \tan\beta$
$\Longrightarrow h^0, H^0, H^\pm, A^0$

At tree level:

$$M^2_{h^0,H^0} = \frac{1}{2}\left\{M_A^2 + M_Z^2 \mp \left[(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta\right]^{1/2}\right\}$$

$$M^2_{H^\pm} = M_W^2 + M_A^2$$

$$\Longrightarrow \quad \begin{aligned} M_{h^0} &< M_Z |\cos 2\beta| \\ M_{H^0} &> M_Z \\ M_{H^\pm} &> M_W \end{aligned}$$

Radiative Corrections

$$M^2_{h^0} \simeq M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2} log \frac{\tilde{m}^2_{t_1} \tilde{m}^2_{t_2}}{m_t^4}$$

The SU(5) Model (Georgi-Glashow)

From group theory (e.g. Slansky, Physics Repts):

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

$$\begin{aligned} 5 &= (3,1)_{-2/3} + (1,2)_1 && \text{– fundamental rep : } \psi_i \\ 10 &= (3,2)_{1/3} + (\bar{3},1)_{-4/3} + (1,1)_2 \\ 24 &= (8,1)_0 + (3,2)_{-5/3} + (\bar{3},2)_{5/3} + (1,3)_0 + (1,1)_0 && \text{– adjoint rep} \end{aligned}$$

In addition:

antisymmetric tensor $\psi_{ij} = -\psi_{ji}$

$$\downarrow$$

$$\begin{aligned} 5 \times 5 &= 10 + \bar{15} \\ 10 \times 10 &= \bar{5} + \bar{45} + 50 \\ \bar{5} \times 10 &= 5 + 45 \\ 5 \times \bar{5} &= 1 + 24 \end{aligned}$$

Recall:

Dirac equation in electromagnetic field:

$$(i\gamma^\mu \partial_\mu - q\gamma^\mu A_\mu(x) - m)\,\psi(x) = 0 \quad \rightarrow$$

$$\begin{cases} (i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu(x) - m)\,\Psi(x) = 0 & - \Psi(x) : \textit{electron} \\ (i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu(x) - m)\,\Psi^c(x) = 0 & - \Psi^c(x) : \textit{positron} \ \ (\textit{antiparticle}) \end{cases}$$

$$\rightarrow \Psi^c(x) = i\gamma^2 \Psi^*(x)$$

$$\Longrightarrow \ (\Psi_R)^c = (\Psi^c)_L \equiv \Psi^c_L$$

Then we can write e.g. the quarks of the first family using the $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers and only left-handed 2-component fields:

$$u_L, d_L : (3,2)_{1/3}$$
$$u^c_L : (\bar{3},1)_{-4/3}$$
$$d^c_L : (\bar{3},1)_{2/3}$$

A comparison shows that one family of fermions of the SM can be accommodated in two $SU(5)$ irreps (or three if ν_R exists):

$$\bar{5} : (\Psi^i)_L = (d^{c1} d^{c2} d^{c3} e^- \; - \nu_e)_L$$

or

$$5 : (\Psi_i)_R = (d_1 \; d_2 \; d_3 \; e^+ \; - \nu_e^c)_L$$

and

$$10 : (\chi_{ij})_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^{c3} & -u^{c2} & u_1 & d_1 \\ & 0 & u^{c1} & u_2 & d_2 \\ & & 0 & u_3 & d_3 \\ & & & 0 & e^+ \\ & & & & 0 \end{pmatrix}$$

The combination $\bar{5}$ and 10 is anomaly free.

$SU(5)$ Generators

$\{\lambda^a\}$, $a = 1, 2, ..., 24$ is a set of 24 $(5^2 - 1)$ unitary, traceless 5×5 (fundamental rep) matrices with normalization:

$$Tr(\lambda^a \lambda^b) = 2\delta^{ab}$$

satisfying the commutation relations:

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = iC^{abc}\frac{\lambda^c}{2}, \quad C^{abc} \quad \text{– } \textit{structure constants}$$

i.e. generalized Gell-Mann matrices.

Examples:

$$\lambda^a = \begin{bmatrix} & & & 0 & 0 \\ & \lambda^a & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad , a = 1, 2, ..., 8$$

$\hookrightarrow$ Generators of $SU(3)$ - Gell-Mann matrices
(the corresponding gauge bosons are the gluons)

$$\lambda^{9,10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \multicolumn{2}{c}{\sigma_{1,2}} \\ 0 & 0 & 0 & & \end{bmatrix}, \quad \lambda^{11} = diag(0,0,0,1,-1)$$

$\hookrightarrow$ Generators of $SU(2)$ - Pauli Matrices
(the corresponding gauge bosons are the $W^{\pm}, W_3$)

$$\lambda^{12} = \frac{1}{\sqrt{15}} diag(-2,-2,-2,3,3)$$

$\hookrightarrow$ Generator of $U(1)$
(the corresponding gauge boson is B)

$$\lambda^{13} = \begin{bmatrix} & & & 1 & 0 \\ & O & & 0 & 0 \\ & & & 0 & 0 \\ 1 & 0 & 0 & O & \\ 0 & 0 & 0 & & \end{bmatrix}, \quad \lambda^{14} = \begin{bmatrix} & & & i & 0 \\ & O & & 0 & 0 \\ & & & 0 & 0 \\ -i & 0 & 0 & O & \\ 0 & 0 & 0 & & \end{bmatrix}, \quad etc.$$

Charge Quantization

Property of simple non-abelian groups that the eigenvalues of the generators are discrete (recall e.g. $SO(3)$).

- In $SU(5)$ the Q is one (linear combination) of the generators and therefore quantized.
- Since electric charge is an additive quantum number, Q must be some linear combination of the 4 diagonal generators of $SU(5)$ (rank 4).
- Since Q commutes with $SU(3)_c$ generators (2 diagonal; rank 2)

$$\Rightarrow Q = T_3 + \frac{Y}{2} = \frac{1}{2}\lambda^{11} + \frac{c}{2}\lambda^{12}$$

c is determined by comparing the eigenvalues of λ^{12} with the hypecharge Y values of particles in 5

$$\rightarrow \quad c = (5/3)^{1/2}$$

Then the traceless condition:

$$Tr\, Q = 0 \rightarrow 3q_d + q_{e^+} = 0 \;\; !$$

(More) $SU(5)$ Physics

- 1st generation of fermions

$$SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\begin{aligned}
\bar{5} &= (\bar{3},1)_{2/3} + (1,2)_{-1} \\
&= d_L^c + (\nu_e, e^-)_L \\
10 &= (3,2)_{1/3} + (\bar{3},1)_{-4/3} + (1,1)_2 \\
&= (u,d)_L + u_L^c + e_L^+
\end{aligned}$$

- Gauge bosons

$$24 = \underbrace{(8,1)_0}_{\text{gluons}} + \underbrace{(3,2)_{-5/3}}_{X^{-4/3},\,Y^{-1/3}} + \underbrace{(\bar{3},2)_{5/3}}_{X^{4/3},\,Y^{1/3}} + \underbrace{(1,3)_0}_{W^{\pm},\,W_3} + \underbrace{(1,1)_0}_{B}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} & & & X_1 & Y_1 \\ & gluons & & X_2 & Y_2 \\ & G & & X_3 & Y_3 \\ X^1 & X^2 & X^3 & W_3/\sqrt{2} & \\ Y^1 & Y^2 & Y^3 & & -W_3/\sqrt{2} \end{bmatrix} + \frac{B}{\sqrt{60}}\begin{bmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{bmatrix}$$

From covariant derivatives:

$$\mathcal{L}_{int} = - \frac{g_5}{2} G^a_\mu (\bar{u}\gamma^\mu \lambda^a u + \bar{d}\gamma^\mu \lambda^a d) - \frac{g_5}{2} W^i_\mu (\bar{Q}_L \gamma^\mu \tau^i Q_L + \bar{L}\gamma^\mu \tau^i L) - \frac{g_5}{2}\left(\frac{3}{5}\right)^{1/2 *} B_\mu \sum_{all\ fermions} \bar{f}\gamma^\mu Y f + \text{interactions of } X, Ys$$

Note that $g_{strong} = g_{SU(2)} = g_5$

However, $^* \; g_5 \lambda^{12} A^{12}_\mu = \underbrace{g'}_{\text{of SM}} Y B_\mu$

and $Y = (5/3)^{1/2} \lambda^{12} \quad \rightarrow \quad g' = (3/5)^{1/2} g_5$

$$\rightarrow \quad \sin^2\theta_W = \frac{g'^2}{g^2_{SU(2)} + g'^2} = \frac{3}{8} \quad !$$

Spontaneous Symmetry Breaking

The first symmetry breaking $SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$ is achieved by introducing a 24-plet of scalars $\Phi(x)$, with potential

$$V(\Phi) = -m_1^2(Tr\Phi^2) + \lambda_1(Tr\Phi^2)^2 + \lambda_2(Tr\Phi^4)$$

for $\lambda_1 > -7/30, \ \lambda_2 > 0$

$$< \Phi >= V \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ & & & & -3/2 \end{pmatrix} \quad L - F.Li$$

where $V^2 = m_1^2/[15\lambda_1 + (7/2)\lambda_2]$

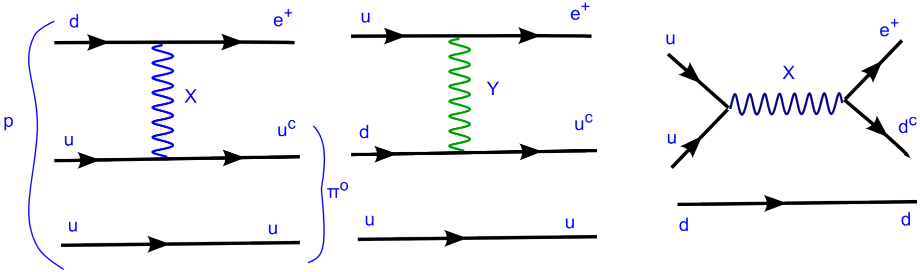
The gauge bosons X, Y obtain mass:

$$m_X^2 = m_Y^2 = \frac{25}{8} g_5^2 V^2$$

Examining further the interactions of X, Y we find:

$$\mathcal{L}_{XYint} = - \frac{g_5}{2}\left[X^{-}_{\mu,\alpha}\left(\bar{d}^{\alpha}_R\gamma^{\mu}e^c_R + \bar{d}^{\alpha}_L\gamma^{\mu}e^c_L + \epsilon^{\alpha\beta\gamma}\bar{u}^c_{L\gamma}\gamma^{\mu}u_{L\beta}\right) + h.c.\right]$$
$$- \frac{g_5}{2}\left[Y^{-}_{\mu,\alpha}\left(\bar{d}^{\alpha}\gamma^{\mu}\nu^c_R + \bar{u}^{\alpha}_L\gamma^{\mu}e^c_L + \epsilon^{\alpha\beta\gamma}\bar{u}^c_{L\beta}\gamma^{\mu}d_{L\gamma}\right) + h.c.\right]$$

leading to proton decay via diagrams such as:



with strength $\sim g_5^2/m^2_{X,Y}$

$$\rightarrow T_p \simeq \frac{m_X^4}{g_5^4 m_p^5}$$

limits $T_p \geq 10^{31} y \rightarrow m_{X,Y} \geq 10^{15}\ GeV$

Gauge Hierarchy Problems

The second breaking in $SU(5)$, i.e. $SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{e/m}$ is due to a 5-plet of scalars. Then the complete potential is:

$$V = V(\Phi) + \underbrace{V(H)}_{\text{5-plet}} + V(\Phi, H)$$

with

$$\begin{aligned} V(H) &= -m_2^2 H^\dagger H + \lambda_3 (H^\dagger H)^2 \\ V(H,\Phi) &= \alpha (H^\dagger H)(Tr\Phi^2) + \beta H^\dagger \Phi^2 H \end{aligned}$$

In turn the vev of Φ changes a bit:

$$< \Phi >= V \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -\frac{3}{2} - \frac{\epsilon}{2} & \\ & & & & -\frac{3}{2} + \frac{\epsilon}{2} \end{pmatrix}$$

when

$$< H >= \frac{\upsilon}{2} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

for an appropriate range of parameters.
In turn, the gauge boson (X, Y) masses are superheavy:

$$m_X^2 \simeq m_Y^2 = \frac{25}{8} g_5^2 V^2$$

and

$$W, Z: \quad m_W^2 \simeq \frac{g^2 \upsilon^2}{4}(1+\epsilon), \quad m_Z^2 \simeq \frac{g^2 \upsilon^2}{4 \cos^2 \theta_w}$$

- To keep the two scales in the theory $\rightarrow \epsilon \sim 10^{-28}$!
- The triplet in the H 5-plet can mediate proton decay. Therefore should be superheavy, while the doublet has electroweak scale mass.

Use of Renormalization Group Equations

Our picture is that at scales above V we have a gauge invariant $SU(5)$ theory, which at $\sim V$ breaks down spontaneously to the SM $SU(3)_c \times SU(2)_L \times U(1)_Y$.

Then the evolution of the three gauge couplings g_3, g_2, g_1 is controlled by the corresponding β-functions:

$$\frac{dg_i}{dt} = \beta_i(g_i), \quad i = 3, 2, 1$$

with $t = ln\mu$.
Specifically:

$$\beta_3(g_3) = -\frac{g_3^3}{16\pi^2}\left(11 - \frac{2}{3}N_f - \frac{N_H}{6}\right),$$

where N_f is the number of 4-component colour triplet fermions and N_H the number of colour triplet Higgs bosons.

$$\beta_2(g_2) = -\frac{g_2^3}{16\pi^2}\left(\frac{22}{3} - \frac{2}{3}N_f - \frac{N_H}{6}\right)$$

$$\beta_1(g_1) = +\frac{2}{3}N_f\frac{g_1^3}{16\pi^2} + Higgs\ boson\ contr.$$

In addition we have the boundary condition:

$$g_3(M_X) = g_2(M_X) = g_1(M_X)\left(= g'\sqrt{\frac{5}{3}}\right) = g_5$$

We find:

$$M_X = 2.1 \times 10^{14} \times (1.5)^{\pm 1}\left(\frac{\Lambda_{\bar{MS}}}{0.16GeV}\right)$$

And follows:

$$\sin^2\theta_W(M_W) = 0.214 \pm 0.003 \pm 0.006 ln\left(\frac{0.16GeV}{\Lambda_{\bar{MS}}}\right)$$

Experiment: $\sin^2\theta_W = 0.23149 \pm 0.00017$

Fermion Masses in $SU(5)$

Introduce Yukawa couplings:

$$f_1 10_f 10_f 5_H + f_2 10_f \bar{5}_f \bar{5}_H$$

$$< 5_H >= \begin{pmatrix} 0 & 0 & 0 & 0 & \upsilon/\sqrt{2} \end{pmatrix}$$

$$\rightarrow \frac{\upsilon}{\sqrt{2}} f_1 \bar{u} u + \frac{\upsilon}{\sqrt{2}} f_2 (\bar{d} d + \bar{e} e)$$

i.e. $"m_{e/\mu/\tau}" = "m_{d/s/b}"$ respectively

holding at mass scales that $SU(5)$ is a good symmetry, subject to significant renormalization corrections

$$\rightarrow m_b/m_\tau \simeq 3 \quad (!) \quad at \quad \mu = \mu_{th} \sim 10 GeV.$$

$$but \quad \underbrace{m_\mu/m_e}_{\simeq 200} = \underbrace{m_s/m_d}_{\simeq 20}$$

which can be improved to $m_\mu/m_e = 9 m_s/m_d$ by introducing a 45-plet of scalars.

The Supersymmetric $SU(5)$

Motivation: Try to solve the hierarchy problem.

Recall that tree level parameters were tuned to an accuracy 10^{-26} to generate $M_x/m_w \sim 10^{12}$ in the scalar potential of $SU(5)$.

This relation is destroyed by renormalization effects. It has to be enforced order by order in perturbation theory.

Supersymmetry *can* solve the technical problem. If it is exact, the mass parameters of the potential (in fact the whole superpotential) do not get renormalized (non-renorm. theorem).

When SUSY is broken, corrections are finite and calculable ($\propto \Delta m^2$ square mass splitting in supermultiplet).

RGEs estimation of the GUT scale and proton lifetime

- gauge bosons the same
- larger number of spinors and scalars

$\rightarrow$ smaller - in absolute value - $\beta-$ function and therefore slower variation of the asymptotically free couplings

$$\Longrightarrow \qquad \begin{cases} M_{x,susy} \simeq 10^{16} - 10^{17}\, GeV \\ \sin^2\theta_w(m_w) = 0.232 \quad (!) \end{cases}$$

Proton decay is out of reach with usual gauge boson exchange.

There are new diagrams (based on dimension 5 operators - dressed by gauginos -) that become dominant.

The resulting proton lifetime is compatible with presently known bounds.

An important outcome is that the dominant decay mode is different from the $SU(5)$ model and is:

$$p \quad \rightarrow \quad K^+ \quad v_\mu$$

(Due to the anti-symmetry in the colour index that appears, the resulting operators are flavour non-diagonal)

Attempts towards Gauge - Yukawa Unification

GUTs can also relate Yukawa couplings among themselves and might lead to predictions

e.g. in $SU(5)$: successful m_τ/m_b

In SO_{10} all elementary particles of each family (both chiralities and ν_R) are in a commmon 16-plet representation.

Natural gradual extension

Attempt to relate the couplings of the two sectors

$\hookrightarrow$ Gauge - Yukawa Unification

Searching for a symmetry is needed - one that relates fields with different spins

$\hookrightarrow$ Supersymmetry Fayet

BUT $N = 2$

GYU - functional relationship derived by some principle.

In:

- Superstrings
- Composite models

In principle such relations exist.

In practice both have more problems than the SM.

Attempts to relate gauge and Yukawa couplings:

- Requiring absence of quadratic divergencies (Decker + Pestieu, Veltman)

$$\begin{aligned} m_e^2 & +m_\mu^2 + m_\tau^2 \\ + & \quad 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) \\ = & \quad \frac{3}{2} m_w^2 + \frac{3}{4} m_z^2 + \frac{3}{4} m_H^2 \end{aligned}$$

- Spontaneous symmetry breaking of SUSY via F-terms

$$\sum_J (-1)^{2J} (2J+1) m_J^2 = 0$$

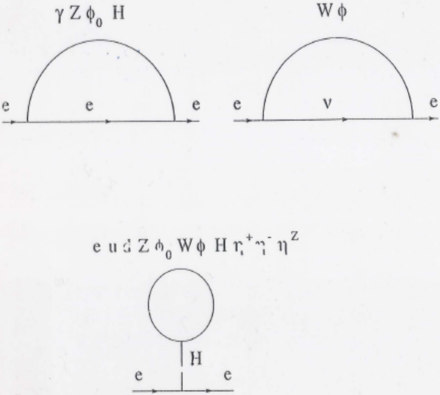


Figure 1

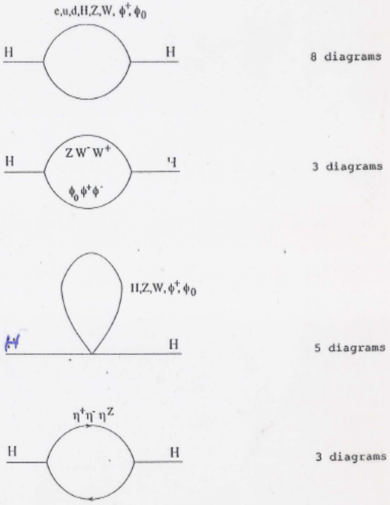


Figure 2

Veltman

Dimensional regularization: quadratic divergencies manifest themselves as pole singularities in $d = 2$

In order to make them vanish, one imposes the condition:

$$\begin{aligned} & \frac{1}{4} f(d) \left[m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) \right] \\ = \quad & \frac{d-1}{2} m_w^2 + \frac{d-1}{4} m_z^2 + \frac{3}{4} m_H^2, \end{aligned}$$

where $f(d) = Tr[1]$

Veltman chose $f(d) = 4$ and using SUSY arguments he put $d = 4$

Osland + Wu using point splitting reg

Jack + Jones + Roberts using DRED

found the same relation

Veltman '81: Requiring absence of quadratic divergencies, he found:

$$\begin{aligned} & m_e^2 + m_\mu^2 + m_\tau^2 \\ + \quad & 3(m_u^2 + m_d^2 + m_c^2 + m_s^2 + m_t^2 + m_b^2) \\ = \quad & \frac{3}{2} m_w^2 + \frac{3}{4} m_z^2 + \frac{3}{4} m_H^2 \end{aligned}$$

- For $m_H^2 << m_w^2 \quad \rightarrow \quad m_t = 69 GeV$
- For $m_H^2 = m_w^2 \quad \rightarrow \quad m_t = 77.5 GeV$

  $\vdots$
- For $m_H^2 = (316 GeV)^2 \quad \rightarrow \quad m_t = 174 GeV$

Ferrara, Girardello and Palumbo considering the spontaneous breaking of a SUSY theory found:

$$\sum_J (-1)^{2J} (2J+1) m_J^2 = 0$$

Chaichian et al

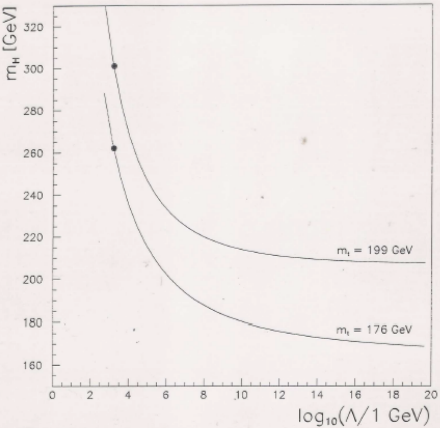


Figure 1: Higgs mass m_H as a function of the scale Λ where cancellation of quadratic divergences is assumed. The bullets denote the intersection points at which the quadratic corrections Δm_H (cf. Eq.(5)) equal the physical mass m_H.

Cancellation of Quadratic Divergencies

Inami - Nishino - Watamura

Deshpande - Johnson - Ma

SUPERSYMMETRY

$\hookrightarrow$ Unique solution?

Standard Model

- Pendleton - Ross infrared fixed point:

For strong α_3, i.e. $\alpha_1 = \alpha_2 = 0$

$$\frac{\mathrm{d}\alpha_3}{\mathrm{d}t} = -7\alpha_3^2$$

$$\frac{\mathrm{d}Y_t}{\mathrm{d}t} = -\frac{Y_t}{4\pi}\left(8\alpha_3 - \frac{9}{2}Y_t\right), \quad Y_t = \frac{h_t^2}{4\pi}$$

$$P - R \quad : \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{Y_t}{\alpha_3}\right) = 0 \quad \rightarrow \quad Y_t = \frac{2}{9}\alpha_3$$

$$\rightarrow \quad m_t^{P-R} = \sqrt{\frac{8\pi}{9}\alpha_3}\, v \quad \sim 100\, GeV$$

In the reduction scheme same result is obtained by the requirement that the system is described by a single coupling theory with a renormalized power series expansion in α_3.

●● Infrared Quasi - fixed point:

Vanishing $\beta-$ function for Y_t

$$\rightsquigarrow \quad \frac{9}{2} Y_t^{Q-f} = 8\alpha_3$$

$$\rightsquigarrow \quad m_t^{Q-f} = \sqrt{8} m_t^{P-R} \sim 280\, GeV$$

⋆ *Quasi - fixed point would also become an exact fixed point if* $\beta_3 = 0$

SUSY S.M.

- Pendleton - Ross:

$$\frac{d\alpha_3}{dt} = -3\alpha_3^2$$

$$\frac{dY_t}{dt} = Y_t\left(\frac{16}{3}\frac{\alpha_3}{4\pi} - 6Y_t\right)$$

$$\frac{d}{dt}\left(\frac{Y_t}{\alpha_3}\right) = 0 \quad \rightarrow \quad Y_t^{susyP-R} = \frac{7}{18}\alpha_3$$

$$m_t^{susyP-R} = \sqrt{\frac{7}{18}4\pi\alpha_3}\, v \sin\beta \quad \sim \quad 140\, GeV \sin\beta$$

$$\tan\beta = \frac{v_u}{v_d}$$

- • Quasi - fixed point:

$$Y_t^{susyQ-f} = \frac{16}{18}\frac{\alpha_3}{4\pi}$$

$$m_t^{susyQ-f} \sim 200\, GeV \sin\beta$$

Quasi - fixed point is reached if h_t becomes strong at scales $\mu = 10^{14} - 10^{19}\, GeV$

Summary

- Pendleton - Ross infrared fixed point

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{Y_t}{\alpha_3}\right) = 0 \quad \rightarrow \quad m_t \sim 100\,GeV$$

(Divergent (!) in 2 - loops, Zimmermann)

- Infrared quasi - fixed point (Hill)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(Y_t\right) = 0 \quad \rightarrow \quad m_t \sim 280\,GeV\,, \quad Y_t \equiv \frac{h_t^2}{4\pi}$$

- SUSY Pendleton - Ross

$$m_t \simeq 140\,GeV \sin\beta \quad , \tan\beta = \frac{v_u}{v_d}$$

- SUSY Quasi - fixed point (Berger et al, Carena et al)

$$m_t \simeq 200\,GeV \sin\beta$$

(If $\tan\beta > 2 \rightarrow m_t^{I.R.} \geq 188\,GeV$, Kubo et al)

We attempt to reduce further the parameters of GUTs searching for renormalization group invariant relations among GUT's couplings holding beyond the unification scale