1. Consider the n-dimensional sphere $S^n : x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = 1$
Construct the (U_N, χ_N), (U_S, χ_S) charts, similar to the ones that we constructed for the 2-dimensional sphere S^2

2. Consider the ellipsis $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Construct the charts (U_Θ, χ_Θ) and (U_N, χ_N) and compute the transition maps between them
Construct (U_S, χ_S).

3. Construct a chart for the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{z^2} = 1$

4. Construct a chart for the hyperboloid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} - \frac{x_3^2}{c^2} = 1$

5. Construct a spherical coordinate system (χ, θ, φ) for S^3, the 3-dimensional sphere $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$.

Generalize this construction for S^n.

6. Consider the 2d manifold given by $x_3 = x_1^2 - x_2^2$

(α) Show that $x_1 = \frac{1}{2}(u+v)$ $x_2 = \frac{1}{2}(u-v)$ $x_3 = uv$ is a coordinate system (u, χ_u) of this manifold. Does it make an atlas for the manifold?

(β) Construct a (u_θ, χ_θ) chart using spherical coordinates in $\mathbb{R}^3$ s.t. $\chi_\theta : (x_1, x_2, x_3) \mapsto (\theta, \varphi)$. Compute the transition map $\chi_u \circ \chi_\theta^{-1}$