1) Consider the energy-momentum tensor (EMT) of a perfect fluid

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p \eta^{\mu\nu}$$

where $u_\mu u^\mu = -1$ its 4-velocity. Compute $u_\nu \partial_\mu T^{\mu\nu}$, and show that $\partial_\mu T^{\mu\nu} = 0$ implies that

$$\partial_\mu (\rho u^\mu) + p\, \partial_\mu u^\mu = 0$$

In the non-relativistic limit $|v^i| \ll 1$, and $p \ll \rho$, show that

$$\partial_t \rho + \vec{\nabla} (\rho \vec{v}) = 0$$

Then, use the projection operator $P^\mu{}_\nu = \delta^\mu{}_\nu + u^\mu u_\nu$ to compute

$$P^\sigma{}_\nu \partial_\mu T^{\mu\nu} = (\rho + p) u^\mu \partial_\mu u^\sigma + \partial^\sigma p + u^\sigma u^\mu \partial_\mu p$$

In the non relativistic limit, show that $\partial_\mu T^{\mu\nu} = 0$ implies that

$$\rho\left(\partial_t \vec{v} + (\vec{v} \cdot \vec{\nabla})\,\vec{v}\right) = -\vec{\nabla} p$$

(See Carroll p 36, Misner et al p 152)

↳ see that for the atmosphere $p \sim 10^{-12} \rho$