The radial nul geodesics are solutions to

$$\frac{dt}{dr} = \pm\left(1 - \frac{2}{r}\right)^{-1}$$

which gives the direction of light cones.

In[•]:=
$$\left\{ \int \frac{1}{1 - \frac{2}{r}}\, d\!r, \ \frac{1}{2\,M} \int \frac{1}{1 - \frac{2\,M}{r}}\, d\!r \right\}$$

Out[•]=
$$\left\{ r + 2\,\text{Log}[-2 + r], \ \frac{r + 2\,M\,\text{Log}[-2\,M + r]}{2\,M} \right\}$$

In[•]:=
```
t [r_, t0_] := r + 2 Log[ r/2 - 1] + t0 ;

ti[r_, t0_] := r + 2 Log[- r/2 + 1] + t0 ;

dt[r_] := 1/(1 - 2/r) ;(* gives slope *)
```
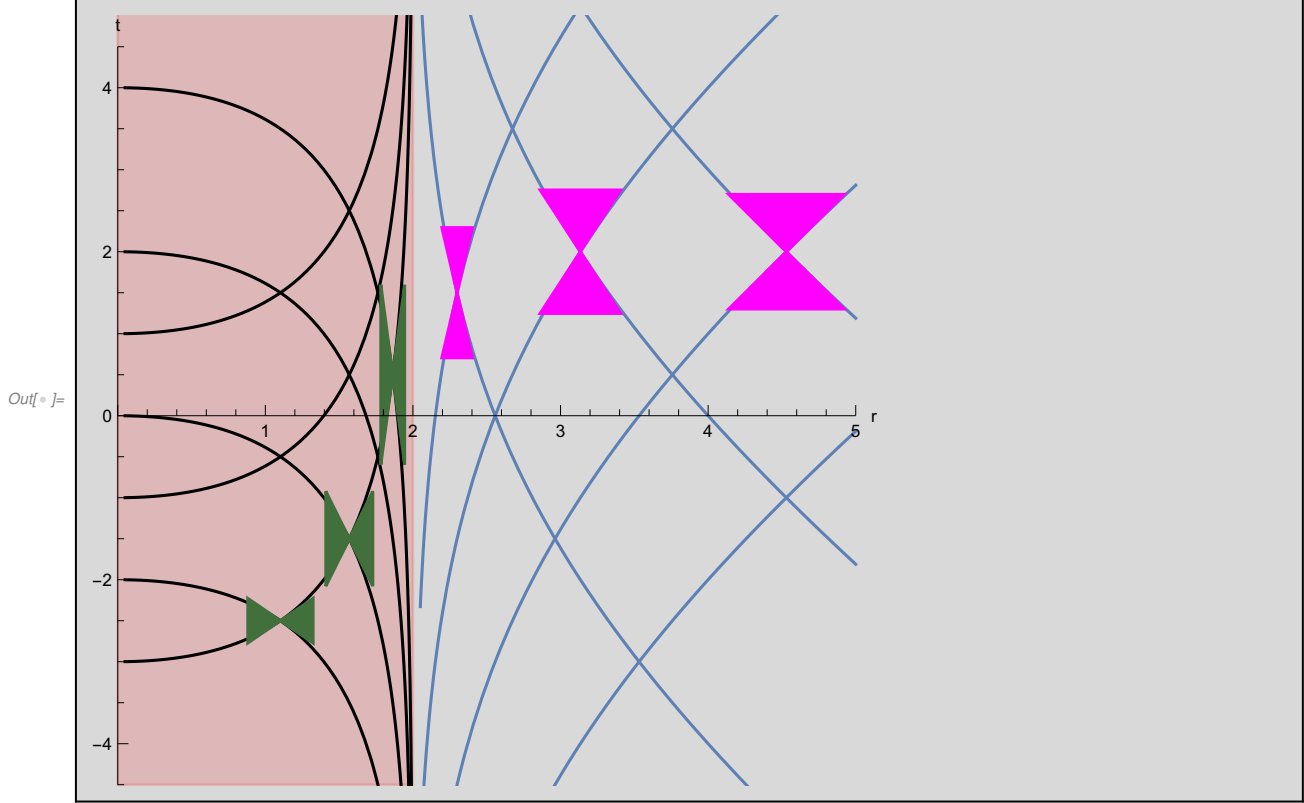
Create a Light Cone graphics object:

```
lightCone[x0_, y0_, len_, slope_, color_] := Module[
   {x1, y1, x2, y2, x3, y3, x4, y4, θ, cone, l},
   l = Abs[len];
   If[slope > 0,
    θ = ArcTan[slope],
    θ = ArcTan[slope] + π
   ]; (* ArcTan gives  -π/2 ≤ θ ≤ π/2 *)
   x1 = x0 + l Cos[θ]; y1 = y0 + l Sin[θ];
   x2 = x0 - l Cos[θ]; y2 = y0 + l Sin[θ];
   x3 = x0 - l Cos[θ]; y3 = y0 - l Sin[θ];
   x4 = x0 + l Cos[θ]; y4 = y0 - l Sin[θ];
   If[slope > 0,
    cone = Polygon[{{x1, y1}, {x2, y2}, {x0, y0}, {x4, y4}, {x3, y3}, {x0, y0}}],
    cone = Polygon[{{x1, y1}, {x4, y4}, {x0, y0}, {x2, y2}, {x3, y3}, {x0, y0}}]
   ];
   Graphics[{color, cone}]
  ];
Show[{lightCone[0, 0, 1, 1, Red], lightCone[2, 3, 1, -1, Blue]}];
```

In[•]:=
```
rmin =  0.0; rmax = 5.0; rS = 2.0;
tmin = -4.5; tmax = 4.5;
g0 = Graphics[{Opacity[0.15], Red, Rectangle[{0, tmin}, {rS, tmax + 1}]}];
g1 = Plot[
   Table[ t[r, t0], {t0, {0.0,  3, -3, -6}}], {r, 2.05, rmax}(*,PlotStyle→{Magenta}*)
```

```
  ];
g2 = Plot[
   Table[-t[r, t0], {t0, {0.0, -4, -7    }}], {r, 2.05, rmax}(*,PlotStyle→{Magenta}*)
  ];
(*Create light cones: *)
t01 =  3.0;
t02 = 0.0; rp = r /. FindRoot[t[r, t01] == -t[r, t02], {r, 4.2}];
tp = t[rp, t01];
g4 = lightCone[rp, tp, 0.8, dt[rp], Magenta];
t01 =  0.0;
t02 = -4.0; rp = r /. FindRoot[t[r, t01] == -t[r, t02], {r, 4.2}];
tp = t[rp, t01];
g5 = lightCone[rp, tp, 0.8, dt[rp], Magenta];
t01 = -3.0;
t02 = -7.0; rp = r /. FindRoot[t[r, t01] == -t[r, t02], {r, 4.2}];
tp = t[rp, t01];
g6 = lightCone[rp, tp, 0.8, dt[rp], Magenta];
(*Inside rS: *)
h1 = Plot[
   Table[ ti[r, t0], {t0, {-2, 0.0, 2, 4}}], {r, 0.05, 1.99}, PlotStyle → {Black}
  ];
h2 = Plot[
   Table[-ti[r, t0], {t0, {-1.0, 1.0, 3.0}}], {r, 0.05, 1.99}, PlotStyle → {Black}
  ];
(*Create light cones: *)
color = RGBColor["#42703d"];
t01 =  4.0;
t02 = 3.0; rp = r /. FindRoot[ti[r, t01] == -ti[r, t02], {r, 1.1}];
tp = ti[rp, t01];
h3 = lightCone[rp, tp, 1.10, dt[rp], color];
t01 =  0.0;
t02 = 3.0; rp = r /. FindRoot[ti[r, t01] == -ti[r, t02], {r, 1.1}];
tp = ti[rp, t01];
h4 = lightCone[rp, tp, 0.60, dt[rp], color];
t01 = -2.0;
t02 = 3.0; rp = r /. FindRoot[ti[r, t01] == -ti[r, t02], {r, 1.1}];
tp = ti[rp, t01];
h5 = lightCone[rp, tp, 0.35, dt[rp], color];
Show[g0, g1, g2, g4, g5, g6, h1, h2, h3, h4, h5,
 PlotRange → {{rmin, rmax}, {tmin, tmax}},
 AspectRatio → 1, Axes → True, AxesLabel → {"r", "t"}]
```

Out[⁃]=

In[⁃]:= `Plot[10 - r - 2 Log[Abs[`$\frac{r}{2}$`- 1]], {r, 0.1, 4}]`

Out[⁃]=